

# Hybrid Fire Testing

## Discussion on stability and implementation of a new method in a virtual environment

Ana Sauca

*Department of Urban and Environmental Engineering, University of Liège, Liège, Belgium*

Thomas Gernay

*Department of Civil Engineering, Johns Hopkins University, Baltimore, Maryland, USA*

Fabienne Robert

*CERIB, Fire Testing Center, Epervan, France*

Nicola Tondini

*Department of Civil, Environmental and Mechanical Engineering, University of Trento, Trento, Italy*

Jean-Marc Franssen

*Department of Urban and Environmental Engineering, University of Liège, Liège, Belgium*

### ABSTRACT

**Purpose** – The purpose of this paper is to propose a method for Hybrid Fire Testing (HFT) which is unconditionally stable, ensures equilibrium and compatibility at the interface and captures the global behavior of the analyzed structure. HFT is a technique that allows assessing experimentally the fire performance of a structural element under real boundary conditions that capture the effect of the surrounding structure.

**Design/methodology/approach** – The paper starts with the analysis of the method used in the few previous HFT. Based on the analytical study of a simple one degree-of-freedom elastic system, it is shown that this previous method is fundamentally unstable in certain configurations that cannot be easily predicted in advance. Therefore, a new method is introduced to overcome the stability problem. The method is applied in a virtual hybrid test on a 2D reinforced concrete beam part of a moment resisting frame.

**Findings** – It is shown through analytical developments and applicative examples that the stability of the method used in previous HFT depends on the stiffness ratio between the two substructures. The method is unstable when implemented in force control on a physical substructure that is less stiff than the surrounding structure. Conversely, the method is unstable when implemented in displacement control on a physical substructure stiffer than the remainder. In multi degrees-of-freedom tests where the temperature will affect the stiffness of the elements, it is generally not possible to ensure continuous stability throughout the test using this former method. Therefore, a new method is proposed where the stability is not dependent on the stiffness ratio between the two substructures. Application of the new method in a virtual HFT proved to be stable, to ensure compatibility and equilibrium at the interface and to reproduce accurately the global structural behavior.

**Originality/value** – The paper provides a method to perform Hybrid Fire Tests which overcomes the stability problem lying in the former method. The efficiency of the new method is demonstrated in a virtual HFT with 3 degrees-of-freedom at the interface, the next step being its implementation in a real (laboratory) hybrid test.

**Keywords:** hybrid fire tests, physical substructure, numerical substructure, stability, control process

## 1. INTRODUCTION

Fire tests are required to understand the behavior of structures exposed to fire. Most generally the standard tests are performed on single elements placed in a furnace and on which fixed (i.e. constant through time) mechanical boundary conditions are applied. Full scale tests are performed on entire structures (e.g. Newman et al., 2000 and Lennon, 2003), but the high cost makes the practice uncommon. However, standard tests fail to capture the effects of the remainder of the structure on the element. Due to thermal expansion of the element exposed to fire, the restraints at the boundary play an important role in the structural behavior because of indirect actions. A promising technique for keeping the advantage of testing only parts of the structure while at the same time considering the global behavior is Hybrid Fire Testing (HFT). In a hybrid test, a single element is physically tested but the boundary conditions are updated during the test to model the real-time effect of the remainder structure on the tested element. Therefore, this technique represents an appealing solution to test structural elements under realistic boundary conditions.

Hybrid testing, which relies on substructuring method, can be used for testing structural elements under different loading conditions caused by wind, blast, impact, fire or seismic events. It has been intensively used in the seismic field, where it was first applied in the early 1970s. Yet, despite being well described in other fields (Saouma and Sivaselvan, 2008), application of hybrid testing in the fire field is far from straightforward. One of the main challenges is related to the necessity to conduct the HFT in real time. This is because, in most element types, the temperature distribution is highly non-uniform and time dependent (this is particularly significant for concrete or timber elements); as a consequence, the time cannot be scaled when performing HFT because the heat transfer through the section of the elements cannot be delayed or paused. Only for metallic elements in which a uniform temperature distribution can develop could it be possible to work in the temperature domain rather than in the time domain, hence using slower heating rates than real time (assuming the effects of creep can be neglected). Another challenge comes from the fact that the effects of thermal expansion generate significant modifications of the interface boundaries. Only a few attempts at HFT have been made in the past on relatively simple systems, which will be briefly discussed hereafter. Laboratories such as BAM in Germany, CERIB in France and NRC in Canada have recently built the experimental facilities that would allow conducting Hybrid Fire Tests (HFT), but the methodology to ensure a proper control of the HFT with compatibility and equilibrium at the interface between the substructures still needs to be developed. The objective of this paper is to present such a methodology, with a particular focus on the conditions for ensuring stability of the process during the HFT regardless of the relative stiffness of the substructures.

## 2. HISTORY AND GENERAL DEFINITIONS IN HFT

### 2.1. History

Early attempts have been done in developing HFT notably in Germany (Kiel, 1990; Hosser et al. 1993, 1995) but the results have not been disseminated in international publications and are not publicly accessible. To the authors' knowledge, the first attempt to perform a HFT that has been

widely reported was made by Korzen et al. (1999, 2002) on a column specimen (tested structure) extracted from a building (surrounding structure). The mode of action between both substructures is exemplified on a one degree-of-freedom (DoF) basis, i.e. the axial column force is measured and adjusted continuously to the model force, which is represented through a – not necessarily constant – stiffness, in displacement control. Robert (2008) and Robert et al. (2010) reported a hybrid fire test on a slab, with three DoFs controlled at the interface, one axial and two rotational. The behavior of the surrounding structure is modelled through an elastic predetermined matrix defined before the test. Mostafaei (2013a, 2013b) performed a hybrid test on a concrete column extracted from a 3D concrete frame, with one axial DoF controlled at the interface. Unlike the previous cases, the surrounding structure was modelled in SAFIR® (Franssen, 2005; Franssen and Gernay, 2017) with some parts exposed to fire. The interaction between the tested structure and surrounding during the hybrid fire test was done manually by the user.

Aside from these HFT on real-scale structural elements, researchers have recently worked on the development of the methodology and presented small-scale experiments. Whyte et al. (2016) developed a new thermomechanical hybrid simulation method by extending the existing mechanical hybrid simulation method in OpenFresco (2016) and OpenSees (2016) through introduction of the temperature DoF and temperature loads. They analyzed a structure consisting of a long-span girder fixed at both ends and supported by a hanger exposed to fire, where the tested structure is a 75 mm long steel dog bone. Schulthess et al. (2016) presented a case study of HFT on a simply supported beam that is connected to a truss element (consisting of a steel coupon) at mid-span. The beam (surrounding structure) remained at ambient temperature throughout the full simulation while the truss element (tested structure) was exposed to thermal loading. Tondini et al. (2016) proposed a static partition solver for HFT based on the FETI algorithm presented by Farhat and Roux (1991). The method was validated in a numerical environment considering a moment-resisting steel frame. In conclusion, the recent developments on HFT have been implemented so far in a numerical environment or on simplified case studies at small dimensions. The only HFT performed on real size elements remain the tests reported by Korzen, Robert and Mostafaei. Therefore, the methodology considered in these tests, which will be referred to here as the “first generation method”, will be examined more in details in Section 3.

## ***2.2. General definitions in HFT***

This Section defines the components that play a role and have to interact during a Hybrid Fire Test (see Figure 1).

1. The physical substructure (PS) represents the structure tested in the furnace. This is the key component of the test, and is selected because the behavior in the fire situation is unknown or cannot be simulated.
2. The numerical substructure (NS) represents the structure whose response is analyzed aside during the HFT. It is the remainder of the structure (surrounding structure), not tested but whose response influences the boundary conditions of the tested element. The response of the NS can be obtained by numerical modeling (e.g. in a finite element (FE) model) or using a predetermined matrix. The use of a FE model is more suitable when parts of the NS are also exposed to fire, so that a nonlinear behavior

of the NS is expected. In contrast, the response of the NS can be accounted for using a predetermined matrix when the NS remains cold. A constant predetermined matrix can be used to capture the behavior of a linear elastic NS. Nonlinear elastic behavior can be modelled by updating the matrix depending on the value of the displacements at the interface.

3. The transfer system between the NS and PS refers to the actuators needed to apply the response from the NS on the PS at each time step. Note that displacements are applied on the PS in a so-called displacement control procedure (DCP) whereas forces are applied in a force control procedure (FCP). The DCP is typically preferable for safety issues related to the actuator stability near collapse unless the PS is very stiff, e.g. stocky shear walls, and control of small displacements becomes unfeasible for the accuracy of the actuator.

4. The data-acquisition system in the furnace refers to the instruments employed to acquire and read data from the test during the process. Examples of such instruments are the displacement transducers and inclinometers and the load measuring devices.

Interaction between the aforementioned components is handled by the control process presented in Figure 1, i.e. the algorithm that ensures the proper run of the HFT with accurate and stable results throughout the process. At frequent intervals (time step  $\Delta t$ ), the displacements or the forces at the interface of the PS are measured using the data-acquisition system and this information is sent to the NS. The reactions (forces or displacements) of the NS at the interface are calculated and then sent back to the PS. The computed reactions are done based on the hybrid fire testing method. There may be an additional delay of time  $\Delta t_p$  requested for the calculation of the NS reaction and for application of the reaction to the PS. The reactions are applied on the PS via the transfer system. The control system compares the output signal with the control signal having the objective to bring the output closer to the control signal. The transfer system is imposing the control signal while the output signal is measured by the instrumentation employed to read the data, i.e. data-acquisition system.

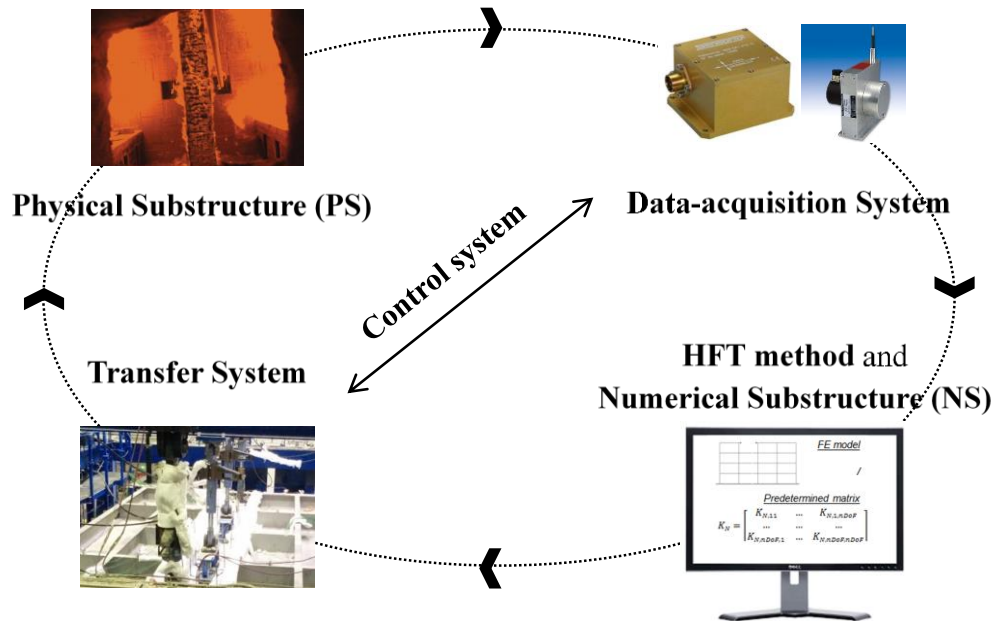


Figure 1. Control process in HFT.

### 3. ANALYSIS OF THE “FIRST GENERATION METHOD” USED IN PREVIOUS HFT

#### 3.1. First generation method applied on a linear 1-DoF system

For the first generation method, when updating the interface forces and displacements, only the characteristics of the NS are considered, disregarding the characteristics of the PS. The consequence of considering only the characteristics of the NS is illustrated here on a simple example. Consider a linear elastic system with a single DoF located at the interface, which is the axial displacement at node 2 (see Figure 2). The temperature in the PS increases with time which induces thermal expansion but, for the sake of simplicity, the stiffness of the PS is assumed to remain constant. The stiffness of the NS also remains constant during the entire duration of the test. The system is composed of two bars, the PS of length  $L_P$  and the NS of length  $L_N$ . The heated PS is defined by the axial stiffness  $K_P$  and thermal coefficient of the material  $\alpha$  whereas the cold NS is characterized by the axial stiffness  $K_N$ . In this virtual HFT, the PS is thus supposed to be placed in a furnace, while the NS is modelled by a predetermined matrix.

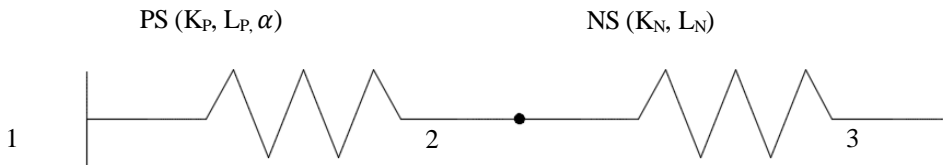


Figure 2. Linear elastic system.

The first generation method using the force control procedure is applied step by step. The equations can be expressed analytically for this simple situation.

- a. First, the analysis of the entire system is performed in order to determine the force and the displacement at the interface between the PS and NS before the start of the fire. In this case, where no external force is applied, the displacement and force at the interface are zero.
- b. The PS is placed in the furnace (in a real HFT) and loaded with the exterior loads and interface conditions, while the NS is modeled aside. Herein the exterior loads, the interface force and displacement are equal to zero.
- c. Heating of the PS starts. In force control procedure, the PS is free to expand, and the displacement is measured. In this example, it yields to the value expressed by Eq. (1).

$$u_P(t_1) = \alpha \cdot L_P \cdot [T(t_1) - T(t_0)] \quad (1)$$

where:

$u_x(t_n)$  is the interface displacement of substructure  $x$  (the subscript can be either P for the PS or N for NS) at time  $t_n$  (i.e. displacement of node 2).

$T(t_n)$  is the temperature of the PS at time  $t_n$ .

$T(t_0)$  is the stress-free reference temperature of the PS.

$n$  is number of the time step.

d. The measured displacement (Eq. (1)) is imposed on the NS. This generates a reaction force that is computed using Eq. (2).

$$F_N(t_1) = K_N \cdot \alpha \cdot L_P \cdot [T(t_1) - T(t_0)] \quad (2)$$

where:

$F_x(t_n)$  is the interface force of substructure  $x$  (P for the PS and N for NS) at time  $t_n$  (i.e. force of node 2).

e. The new reaction force is imposed on the PS (Eq. (3)). Generally, a time delay  $\Delta t_P$  is used to represent the time needed to compute the reaction of the NS and to adjust the force in the jacks, as would be the case for a real HFT. For this example, the NS is modeled by the predetermined matrix, therefore, the time needed to compute the reaction forces is virtually zero. As a consequence, the time delay refers to the time needed to adjust the reactions in the jacks and it cannot be neglected.

$$F_P(t_1 + \Delta t_P) = -K_N \cdot \alpha \cdot L_P \cdot [T(t_1) - T(t_0)] \quad (3)$$

f. The new force induces a new displacement of the PS.

Note: heating of the PS has continued from  $t_1$  to  $t_1 + \Delta t_P$  and from  $t_1 + \Delta t_P$  to  $t_2$  and this induces continuous variation in displacement during computation and while adjusting the force in the jacks.

g. The updated displacement of the PS at the interface  $u_P(t_2)$  is measured at time  $t_2$  (given here by Eq. (4)) and imposed on the NS. This generates a new reaction force  $F_N(t_2)$  given by Eq. (5).

$$\begin{aligned} u_P(t_2) &= \alpha \cdot L_P \cdot T(t_2) + \frac{F_P(t_1 + \Delta t_P)}{K_P} \\ &= \alpha \cdot L_P \cdot \left( [T(t_2) - T(t_0)] - \frac{K_N}{K_P} \cdot [T(t_1) - T(t_0)] \right) \end{aligned} \quad (4)$$

$$F_N(t_2) = K_N \cdot \alpha \cdot L_P \cdot \left( [T(t_2) - T(t_0)] - \frac{K_N}{K_P} \cdot [T(t_1) - T(t_0)] \right) \quad (5)$$

Steps  $e, f, g$  are repeated during the entire hybrid fire test. For future discussion, the ratio between the stiffness of the NS and PS will be referred in this paper as stiffness ratio, i.e.  $R = \frac{K_N}{K_P}$ .

Expanding Eq. (4) and (5), for  $n$  time steps, the displacement at any time step  $t_n$  can be expressed by Eq. (6) and the reaction force generated by the NS by Eq. (7).

$$u_P(t_n) = \alpha \cdot L_P \cdot \sum_{i=0}^{n-1} \{(-R)^i \cdot [T(t_{n-i}) - T(t_0)]\} \quad (6)$$

$$n=1, 2, \dots$$

$$F_N(t_n) = K_N \cdot \alpha \cdot L_P \cdot \sum_{i=0}^{n-1} \{(-R)^i \cdot [T(t_{n-i}) - T(t_0)]\} \quad (7)$$

$n=1, 2, \dots$

The same developments can be made for the displacement control procedure (Sauca, 2016, p. 58). In this case, the measured reaction force can be determined using Eq. (8), while the displacements can be calculated using Eq. (9).

$$F_P(t_n) = -K_P \cdot \alpha \cdot L_P \cdot \sum_{i=0}^{n-1} \left\{ \left(-\frac{1}{R}\right)^i \cdot [T(t_{n-i}) - T(t_0)] \right\} \quad (8)$$

$n=1, 2, \dots$

$$u_N(t_n) = \frac{1}{R} \cdot \alpha \cdot L_P \cdot \sum_{i=0}^{n-1} \left\{ \left(-\frac{1}{R}\right)^i \cdot [T(t_{n-i}) - T(t_0)] \right\} \quad (9)$$

$n=1, 2, \dots$

From the Eq. (6)-(9) it is clear that the results during the HFT, using the first generation method, are influenced by the stiffness ratio  $R$ .

In order to avoid instability, the value in the parenthesis which involves the stiffness ratio should be smaller than 1, i.e.  $R < 1$ , for the force control procedure and  $\frac{1}{R} < 1$  or  $R > 1$  for displacement control procedure. If not, the value tends toward infinity when the number of iteration  $i$  increases, irrespectively of the size of the time steps, and the process becomes unstable.

The conditions for stability in the first generation method are analyzed in the next section.

### 3.2. Conditions for stability

The first generation method is sensitive to the stiffness ratio between the substructures. When the NS is more flexible than the PS, i.e.  $R < 1$ , then the force control procedure FCP is stable, but the displacement control procedure DCP is not. In the case of  $R > 1$ , the DCP is stable, whereas the FCP is not.

Choosing the appropriate procedure between force control and displacement control is not easy due to the following reasons: (i) the stiffness of the PS is changing during the HFT, i.e. the stiffness ratio varies, and (ii) the multiple controlled DoFs at the interface may require different procedures for different DoFs.

(i) *The variation of the stiffness ratio*

Force control procedure

The force control procedure is stable if the NS is more flexible than the PS. Even if the stiffness ratio satisfies the stability condition at ambient conditions  $R < 1$ , it will generally start increasing as a result of the PS being exposed to fire (assuming the NS is kept cold). Instability will occur if the stiffness ratio increases until the critical value of 1. The force control procedure can thus be safely used if the stiffness ratio never exceeds the value of 1 for the entire test duration. The problem is that the evolution of the stiffness of the PS cannot be predicted in a deterministic manner before the test because the purpose of the test is to determine the behavior and characteristics of the PS.

Displacement control procedure

The continuous degradation of the stiffness of the PS during the fire (assuming a constantly increasing temperature), on the other hand, leads to the conclusion that a system which is stable at ambient conditions will remain so during the entire test duration.

(ii) *The multiple controlled degrees-of-freedom*

In general, several DoFs are involved at the interface when a PS is extracted from a wider structure. Whereas one procedure would be required for some of them, the other procedure might be required for the others. It is theoretically possible to use both procedures simultaneously during the same hybrid test, i.e. displacement control for some DoFs and force control for others (Elkhoraibi and Mosalam, 2007). For now, a combined procedure has never been utilized in the fire field. More theoretical research may be needed to prove the feasibility of the combined procedure in HFT and, from the experimental point of view, the ability to control some displacements along with forces may not be available in all facilities.

In order to avoid the instability which might occur prematurely in the hybrid fire tests, a new method needs to be developed which can be applied independently of the stiffness ratio between the substructures.

**3.3. Discussion on the previous hybrid fire tests**

In the previous hybrid tests made on full scale structures, the reactions to be applied on the interface of the PS are computed based on the force control procedure. A posteriori analysis of the configuration of these tests shows that, in all cases, the NS was more flexible than the PS during the hybrid test. Therefore, the adopted procedure was adequate and its application did not result in any stability issue during the tests. The test by Korzen and the one by Mostafei have been performed on column supposed to be part of a moment resisting frame. Because the axial stiffness of a column is very large compared to the stiffness related to a vertical movement in a moment resisting frame (at a position where the column has been removed), the stiffness ratio was much lower than 1 in these tests. Robert presents a hybrid test where the physical substructure is represented by a reinforced concrete slab. Three DoFs were controlled at the interface, one axial DoF and two rotational DoFs. It was considered during the test that the axial DOF is not coupled to the rotational DOFs. The stiffness ratio



at ambient temperature was equal to 0.167 for the axial DOF and 0.756 for the rotational DOFs. These ratios increased during the test since the stiffness of the PS was decreasing due to the fire exposure. For the axial DOF the stiffness ratio at ambient temperature is significantly smaller than 1, but this is not the case for the rotational DOF. The test could be performed to the end but it has to be mentioned that the amplitude of the loads applied to the PS had to be limited in order to cope with the unstable behavior that otherwise appeared.

#### 4. DEVELOPMENT OF A NEW METHOD FOR HFT

The new hybrid methodology aims at the following objectives:

- To be stable independently on the stiffness ratio between the substructures;
- To provide accurate results by ensuring equilibrium of forces and compatibility at the interface during the whole duration of the fire.

The method has been inspired from the finite element tearing and interconnecting method (FETI) (Fahrat and Roux, 1991). During the HFT the interface displacements are controlled, based on the out of balance forces between the substructures. A displacement control procedure is selected in order to reproduce the behavior of the PS until failure, including in a possible phase involving large displacements where the structure becomes unstable.

##### 4.1. Theoretical formulation

The key idea behind the second generation method is the fact that the stiffness of both the PS and the NS need to be considered when relating the forces with the displacements at the interface. This contrasts with the first generation method, employed in all previous hybrid fire tests documented in the literature, in which only the stiffness of the NS was considered (see Eq. (2), where the new force to be imposed at the interface of the PS is computed based on the measured displacement in the furnace and the stiffness of the NS).

This modification will allow the hybrid process to be stable independently on the ratio between the stiffness of the NS and the PS. In other words, it becomes possible for a fire laboratory which works in displacement control to perform a hybrid fire test for example on a concrete column axially restrained ( $R < 1$ ), without the need to change their control system. More important, HFT in which several degrees of freedom are controlled can be conducted with a unique methodology.

In theory, the second generation method can be used in displacement control or in force control. The formulation to combine the stiffness of the PS and the stiffness of the NS differs for the two control procedures.

In the displacement control procedure, the displacement at the interface is imposed to be the same for the PS and the NS. Therefore, the total stiffness of the two subsystems is equal to the sum of their individual stiffness (as in an assembly of two substructures in parallel), see Eq. (10).

$$K = K_N + K_P \quad (10)$$

In the force control procedure, on the other hand, the force at the interface is imposed to be the same for the PS and the NS. Therefore, the total flexibility of the two subsystems is equal to sum of their flexibilities (as in an assembly of two substructures in series), leading to Eq. (11). Note that, in this case, the total stiffness is always smaller than the value of the smallest stiffness.

$$K = \left( \frac{1}{K_N} + \frac{1}{K_P} \right)^{-1} \quad (11)$$

#### 4.2. The new method applied on a linear 1-DoF system in DCP

The linear elastic 1-DoF system presented in the Section 3.1 is analyzed here when applying the new methodology. The analytical expressions are presented for this simple example when the displacement control procedure is applied step by step.

- a. The interface forces and displacements are determined before the start of the test by performing the analysis of the entire structure under ambient temperature conditions.
- b. The PS is placed in the furnace and loaded with the exterior loads and interface displacements, while the NS is modeled aside (Sauca, 2016, pg. 107). Herein the exterior loads, the interface forces and displacements are equal to zero (see Eq. (12)).

$$u(t_0) = 0 \quad (12)$$

- c. The interface equilibrium for the ambient temperature will be restored if needed (Sauca, 2016, pg. 108 – 110). For this example no equilibrium at ambient conditions needs to be restored.
- d. Heating of the PS starts.
- e. The interface displacements of the PS are blocked for the duration of a time step (displacement control procedure) and the reaction forces are measured at the end of the time step. In this example it yields to the value expressed by the Eq. (13).

$$F_P(t_1) = -K_P \cdot \alpha \cdot L_P \cdot [T(t_1) - T(t_0)] \quad (13)$$

- f. The reaction forces of the NS are computed based on the stiffness of the NS sole using Eq. (14). If the NS is heated, the reaction forces vary during one time step. If the NS is kept cold, then the reaction forces is constant during one time step.

$$F_N(t_1) = K_N \cdot u(t_0) = 0 \quad (14)$$

- g. The measured reaction forces of the PS are compared with the computed reaction forces of the NS. Generally, the equilibrium is not ensured due to the fire effect and the out of balance force  $\Delta F$  is evaluated (Eq. (15)).

$$\Delta F(t_1) = F_P(t_1) + F_N(t_1) = -K_P \cdot \alpha \cdot L_P \cdot [T(t_1) - T(t_0)] \quad (15)$$

- h. To restore the equilibrium, the incremental displacement  $\Delta u$  will be calculated using Eq. (16).

$$\begin{aligned}\Delta u(t_1) &= -(K_N + K_P^*)^{-1} \cdot \Delta F(t_1) \\ &= \frac{K_P}{K_N + K_P^*} \cdot \alpha \cdot L_P \cdot [T(t_1) - T(t_0)]\end{aligned}\quad (16)$$

In the calculation process, the stiffness of the PS ( $K_P^*$ ) and the NS ( $K_N$ ) are accounted for. This is the main difference with the first generation method and the most important contribution that ensures stability of the method. In the first generation method, only the stiffness of the NS is considered ( $K_N$ ) while the stiffness of the PS is neglected. Note that  $K_P^*$  is the stiffness of the PS considered in the calculations while  $K_P$  is the real stiffness of the PS measured in the furnace. An estimative value of the stiffness of the PS,  $K_P^*$ , is considered because the real stiffness of the PS,  $K_P$ , during the HFT is generally unknown. Moreover, it is convenient to use a constant value of the stiffness of the PS in the calculations (usually the initial tangent stiffness of the PS). This simplification will imply continuous iterations which will be discussed later.

This step h, and Eq. (16), is the only step where the estimate  $K_P^*$  is considered in the calculations. In the other equations, the real stiffness of the PS ( $K_P$ ) is used since it is referred to the measured values in the furnace during the HFT.

For the studied 1-DoF system, the stiffness considered in the calculations  $K_P^*$  is equal to the real stiffness  $K_P$  of the PS (based on the hypothesis that the stiffness of the PS remains constant while exposed to fire, during this entire exercise).

i. The new calculated displacements (Eq. (17)) are imposed on the PS and NS. The time needed to perform these calculations and to adjust the new displacements in the furnace ( $\Delta t_P$ ) is accounted for. Thus, the displacement is applied on the PS at the time  $t_1 + \Delta t_P$ .

$$u(t_1) = u(t_0) + \Delta u(t_1) = \frac{K_P}{K_N + K_P^*} \cdot \alpha \cdot L_P \cdot [T(t_1) - T(t_0)] \quad (17)$$

j. The new imposed displacements will generate new reaction forces in the PS and NS. For the next time step  $t_2$ , the reaction force of the PS is given by Eq. (18) and the reaction force of the NS is computed using Eq. (19).

$$F_P(t_2) = -K_P \cdot \alpha \cdot L_P \cdot \left( [T(t_2) - T(t_0)] - \frac{K_P}{K_N + K_P^*} \cdot [T(t_1) - T(t_0)] \right) \quad (18)$$

$$F_N(t_2) = \frac{K_N \cdot K_P}{K_N + K_P^*} \cdot \alpha \cdot L_P \cdot [T(t_1) - T(t_0)] \quad (19)$$

k. The incremental displacement  $\Delta u(t_2)$  (Eq. (21)) is computed based on the out of balance force  $\Delta F(t_2)$  (Eq. (20)). This yields to a value of displacement  $u(t_2)$  expressed by Eq.(22).

$$\begin{aligned}\Delta F(t_2) &= F_P(t_2) + F_N(t_2) \\ &= -K_P \cdot \alpha \cdot L_P \cdot \left( [T(t_2) - T(t_0)] - \frac{K_N + K_P}{K_N + K_P^*} \cdot [T(t_1) - T(t_0)] \right)\end{aligned}\quad (20)$$

$$\begin{aligned}\Delta u(t_2) &= -(K_N + K_P^*)^{-1} \cdot \Delta F(t_2) \\ &= \frac{K_P}{K_N + K_P^*} \cdot \alpha \cdot L_P \cdot \left( [T(t_2) - T(t_0)] - \frac{K_N + K_P}{K_N + K_P^*} \cdot [T(t_1) - T(t_0)] \right)\end{aligned}\quad (21)$$

$$u(t_2) = u(t_1) + \Delta u(t_2) = \frac{K_P}{K_N + K_P^*} \cdot \alpha \cdot L_P \cdot [T(t_2) - T(t_0)] \quad (22)$$

Expanding the above equations for  $n$  time steps, the reaction force of the PS can be expressed by the Eq. (23), while the reaction force of the NS is computed using the Eq. (24). Therefore, the computed displacement for  $n$  time steps can be calculated using the Eq. (25).

$$F_P(t_n) = -K_P \cdot \alpha \cdot L_P \cdot \left( [T(t_n) - T(t_0)] - \frac{K_P}{K_N + K_P^*} \cdot [T(t_{n-1}) - T(t_0)] \right) \quad (23)$$

$$n=1, 2, \dots$$

$$F_N(t_n) = \frac{K_N \cdot K_P}{K_N + K_P^*} \cdot \alpha \cdot L_P \cdot [T(t_{n-1}) - T(t_0)] \quad (24)$$

$$n=1, 2, \dots$$

$$u(t_n) = \frac{K_P}{K_N + K_P^*} \cdot \alpha \cdot L_P \cdot [T(t_n) - T(t_0)] \quad (25)$$

$$n=1, 2, \dots$$

The new method is stable, independently on the stiffness ratio between the substructures, as it is expressed by the Eq. (23)-(25). On the contrary, the Eq. (6)-(9) presented in the Section 3.1 show that the stability of the HFT when considering the first generation method depends on the stiffness ratio  $R$ .

Besides being stable, the new method must ensure the interface equilibrium and compatibility. The next discussion concerns the conditions (exemplified on the 1-DoF system) to ensure the equilibrium and compatibility during the HFT when using the new method. The equilibrium condition implies equal absolute values of interface forces, while the compatibility condition implies equal displacement at the interface of the substructures.

The new method ensures the interface compatibility since the computed displacement  $u(t_n)$  (Eq. (25)) is imposed at the interface of the PS and NS. The computed displacement  $u(t_n)$  is imposed on the PS by the time  $t_n + \Delta t_P$ , where  $\Delta t_P$  refers to the time of calculation of the new solution and the time to impose the target displacement on the PS by the actuators. The same displacement is imposed at the interface of the NS by the time  $t_n$  (the delay time is virtually zero in the case of the NS).

The reaction forces of the PS and NS induced by the displacement  $u(t_n)$  (applied on the time  $t_n + \Delta t_P$  for the PS and time  $t_n$  for the NS) needs to be in equilibrium. The equilibrium is ensured

when the out of balance force  $\Delta F(t_n + \Delta t_p)$  at the time  $t_n + \Delta t_p$  (Eq. (26)) is equal to zero. This is possible only when the term of the parenthesis is equal to zero. The value of the parenthesis is dependent on the delay time  $\Delta t_p$  and the stiffness matrix of the PS considered in the calculations, expressed herein as  $K_p^*$ . If the delay time is small enough (tends to 0 s), then the temperature  $T(t_n + \Delta t_p)$  is approximately equal to the temperature  $T(t_n)$ . If the stiffness matrix  $K_p^*$  used in the calculations equals the real stiffness matrix of the PS  $K_p$  then the out of balance force tends to a value of zero and the equilibrium is satisfied.

If the temperature  $T(t_n + \Delta t_p)$  is different than the temperature  $T(t_n)$  (assuming that  $K_p^* = K_p$ ), then the out of balance force is different than zero and the equilibrium is not satisfied. This can occur when  $\Delta t_p$  is excessively large and leads to a significant increase of temperature in the PS from the time  $t_n$  to the time  $t_n + \Delta t_p$ .

$$\begin{aligned} \Delta F(t_n + \Delta t_p) &= F_p(t_n + \Delta t_p) + F_N(t_n) \\ &= -K_p \cdot \alpha \cdot L_p \cdot \left( [T(t_n + \Delta t_p) - T(t_0)] - \frac{K_N + K_p}{K_N + K_p^*} \cdot [T(t_n) - T(t_0)] \right) \end{aligned} \quad (26)$$

$n=1, 2, \dots$

In the case of the 1-DoF linear elastic system, the stiffness of the PS remains constant during the HFT ( $K_p^* = K_p$ ). Therefore, equilibrium is achieved when the delay time  $\Delta t_p$  does not lead to an increase of the temperature in the heated substructure.

In a real HFT, the stiffness of the PS is generally unknown. The initial tangent stiffness may be considered in the calculations as  $K_p^*$  and kept constant during the test. As a consequence, several iterations are needed at each time step to converge to the correct solution. Yet, in a fire test, the evolution of temperatures in the PS cannot be put on hold for the period requested to perform the iterations at every time step. The temperatures keep on increasing during the period of time needed to perform the calculations in the computer and for the testing equipment to apply the corrections of displacements, and this continuously modifies the stiffness of the PS, the restraint forces, etc. Hence, the convergence process is aiming at a state of equilibrium that is constantly changing. As a result, it is not relevant to distinguish between iterations and time steps. Instead, the test can be performed by applying continuously Eq. (25) with a cycling frequency that is as high as possible, which requires computing techniques and testing equipment that has a short response time (hence the advantage of representing the NS by a predetermined matrix). The purpose of the methodology is thus to constantly adapt these displacements to satisfy equilibrium between the substructures throughout the entire test duration (Sauca et al., 2016b).

The analytical expressions developed for the 1-DoF linear system show that the new method is stable while the equilibrium and compatibility at the interface are ensured. At the same time, the new method must reproduce the exact solution at the interface, which we refer as the “correct” solution. For the linear 1-DoF system presented in the Figure the “correct” displacement of the node 2 is given by the Eq. (27). No exterior forces are acting on the system and only the PS is exposed to fire.

$$u(t_n) = \frac{E_p \cdot A_p}{K_N + K_p} \cdot \alpha \cdot [T(t_n) - T(t_0)] \quad (27)$$

$n=1, 2, \dots$

Where:

$E_p$  is the Young modulus of the PS;

$A_p$  is the sectional area of the PS;

The displacement generated by the new method of HFT (Eq. (25)) is equal with the correct solution (Eq. (27)) when the stiffness of the PS considered in the calculations  $K_p^*$  is equal with the real stiffness of the PS  $K_p$ .

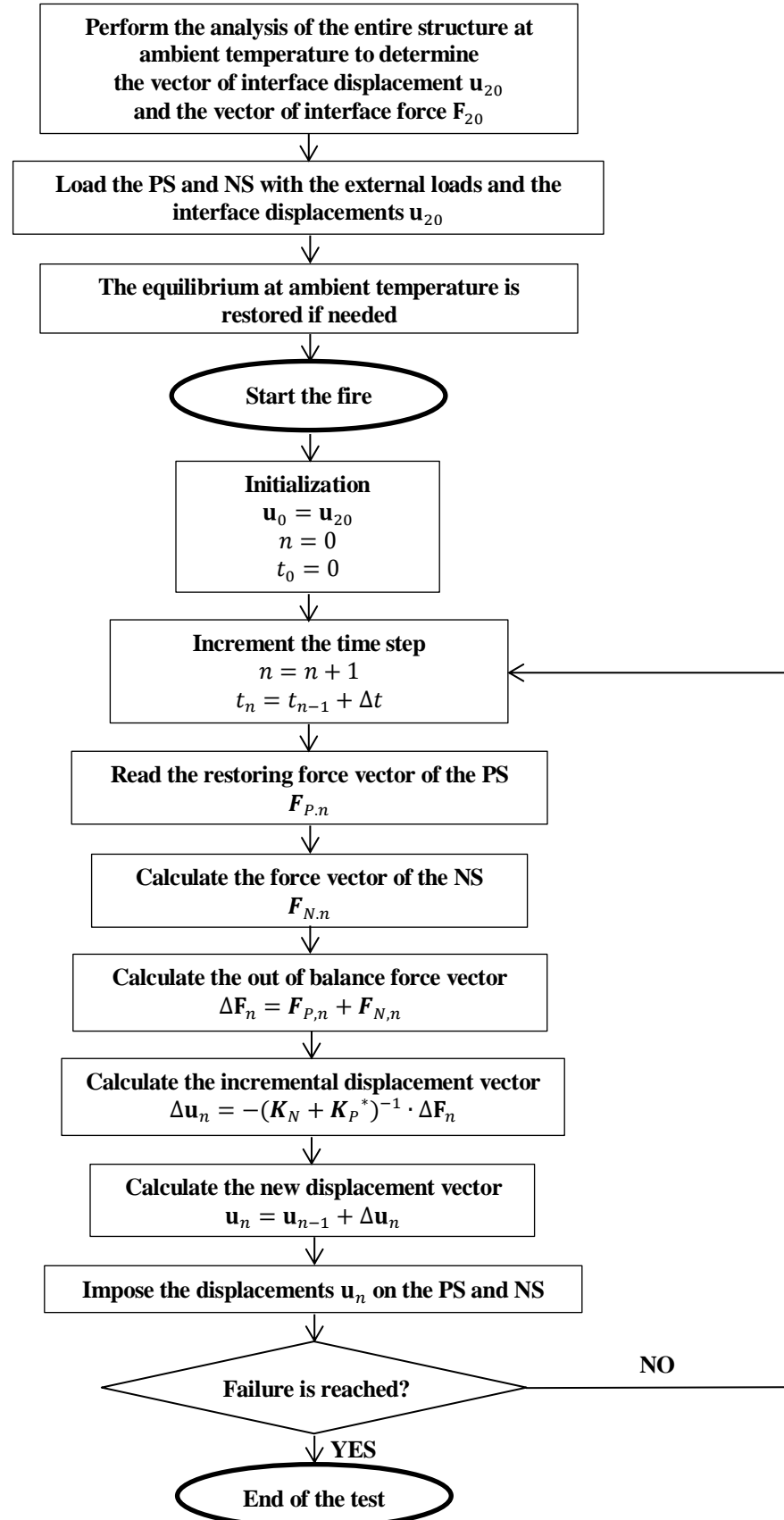
Since continuous iteration process is employed during the HFT, the generated solution will converge to the correct solution.

The new method is stable, able to ensure the interface equilibrium and compatibility, and to reproduce the correct solution at the interface.

#### ***4.3. The new method applied on a general case***

The procedure presented for 1-DoF linear system is depicted in the flowchart given in Figure 3 on a general case, i.e. multiple degrees of freedom at the interface. Moreover, since the stiffness of the exposed substructure degrades due to the fire exposure, continuous iterations are implemented in order to be able to ensure all the objectives of hybrid fire testing, i.e. compatibility, equilibrium and reproduction of the correct solution.

Figure 3. The new method of hybrid fire testing



## 5. APPLICATIONS

### 5.1. Elastic system

In this section a numerical example will be used to illustrate the dependency of the first generation method on the stiffness ratio  $R$  and the improvement provided by the new methodology. The elastic system presented in Figure 2 is analyzed in a virtual environment, meaning that PS and the NS are modeled numerically.

The linear system is considered with the following input data:

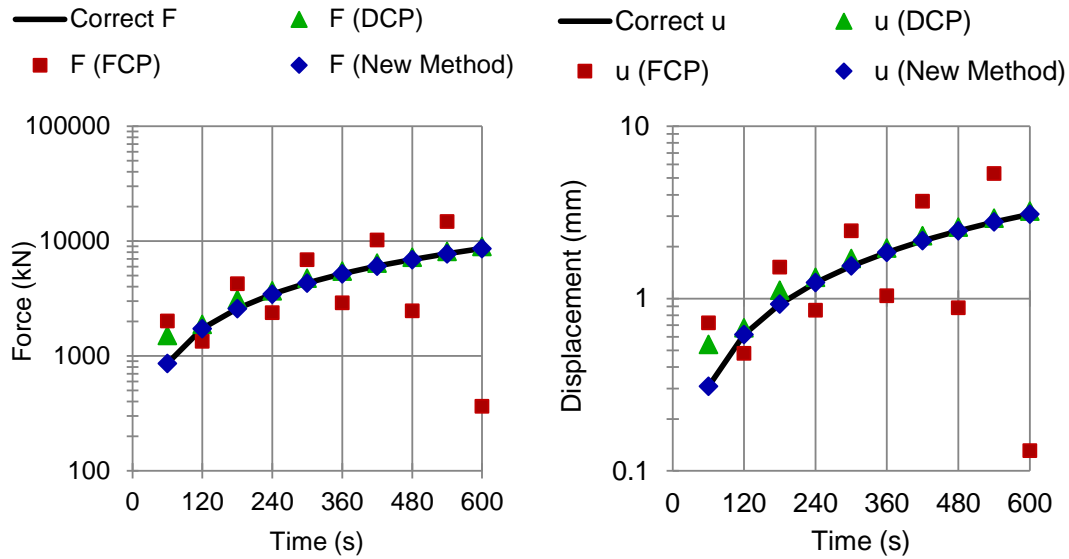
- Coefficient of thermal expansion of the material of the PS:  $\alpha = 12 \times 10^{-6} K^{-1}$ ;
- Young modulus for the PS and NS:  $E_P = E_N = 210\,000\, N/mm^2$
- Sectional area of the PS and NS:  $A_P = A_N = 20\,000\, mm^2$
- The physical substructure is heated at a constant rate of  $0.5\, K/s$ .

For simplicity, the stiffness of the PS is kept constant during the exercise although, it would decrease in a real hybrid test due to the fire exposure.

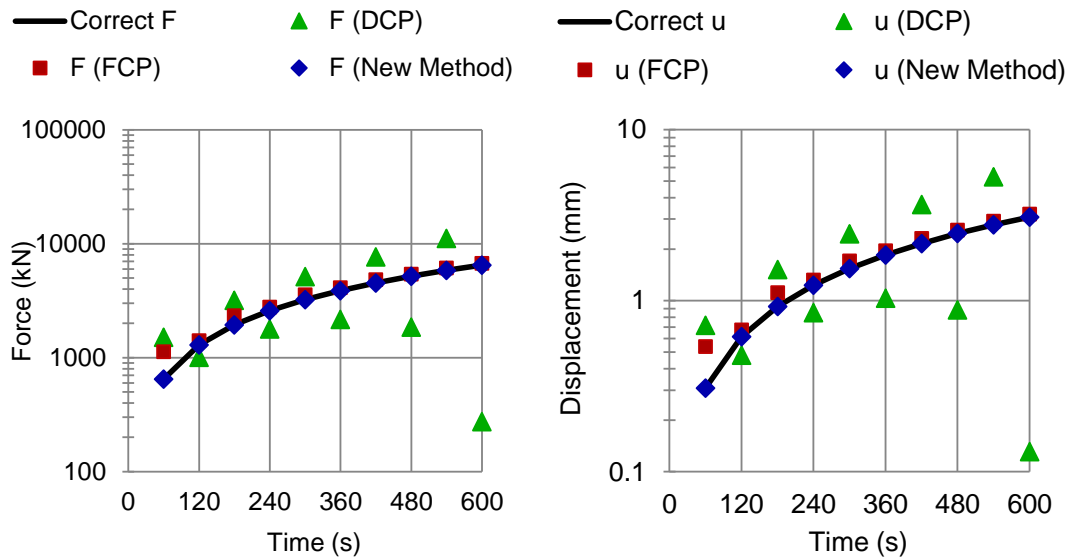
The objective of the exercise is to plot the displacement and force at the interface versus the correct solution. The plots will be done for two different stiffness ratios  $R < 1$  and  $R > 1$  using the first generation method in force control procedure (FCP) and displacement control procedure (DCP), and the new method in displacement control procedure. A length of the PS equal to 1.50 m and a length of the NS equal to 2.00 m yield  $R < 1$  whereas the opposite is obtained if the length of both substructures are swapped.

The evolution of absolute value of displacement and force at the interface is presented for different stiffness ratios in Figure when the time step is equal to 60 s. The calculation time of the solution along with the time needed to apply the solution at the interface of the PS are considered virtually equal to zero. It has to be noted that the scale on the vertical axis is logarithmic and absolute values of displacement and force are used. The logarithmic scale is dictated by the large amplitudes obtained when the process is unstable but this does not show the fact that, in these case, the displacements are oscillating from positive to negative.





a) Stiffness ratio  $R = 1.33$



b) Stiffness ratio  $R = 0.75$

Figure 4. Instability in HFT depending on the stiffness ratio (logarithmic scale).

The reference solution (correct  $u$ , expressed by the Eq. (27)) is the one obtained when the entire system is analyzed, without subdivision. For a stiffness ratio  $R > 1$ , Figure a) shows that the solution diverges from the reference solution when the force control procedure (FCP) is used, while convergence is obtained with displacement control procedure (DCP). Figure b) shows the opposite for a stiffness ratio  $R < 1$ . In contrast, the new method is stable, independently on the stiffness ratio, as can be seen in Figure a) and 4 b).

The above discussion addresses the instability induced by using an inappropriate method. The study of other sources of instabilities, such as the resolution of the data acquisition system, the Hybrid Fire Testing: Discussion on stability and implementation of a new method in a virtual environment

characteristics of the transfer system, the noise effect in the signal or the delay time are also worth being considered but this goes beyond the scope of this paper.

### 5.2. Concrete beam part of a moment resisting frame

The analysis of a 2D reinforced concrete moment resisting frame has been used for a hybrid fire test performed in a virtual environment (both substructures are modelled numerically), using the first generation method and the new method.

In the performed case study, the behavior of the NS is predefined by using a predetermined matrix while the response of the PS is modeled in SAFIR®.

Figure presents the concrete frame, from which the last floor, second span beam, being the only element subjected to the ISO fire, will be taken as the PS.

It has to be understood that although a full geometrically and materially nonlinear analysis is performed in SAFIR® to mimic what the behavior of a real specimen would be in a real furnace, this information is not used in the algorithm of the hybrid test, see section 4.2 and Figure , where the stiffness of the PS is kept constant during the test.

The stiffness of the PS and NS are defined before the virtual hybrid fire test (Sauca, 2016, pg. 146 - 148), using SAFIR®. At ambient temperature, the stiffness ratio for the horizontal DoF is equal to 0.20, 2.53 for the rotation on the left side of the beam respectively 2.48 for the rotation on the right side of the beam.

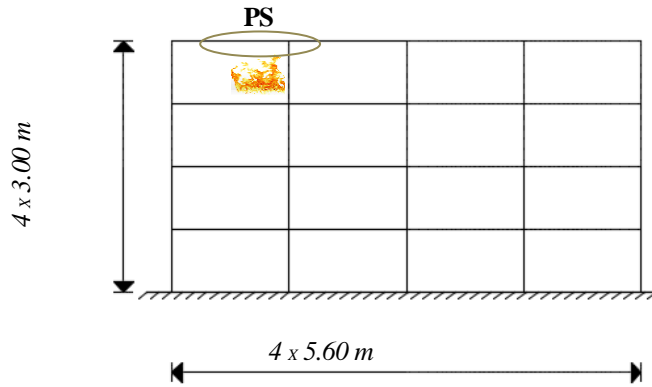


Figure 5. Moment resisting concrete frame.

The configuration of the PS is presented in Figure 6. The concrete beam has a section of 0.25 m x 0.40 m and it will be exposed to fire on three sides only between the supports (5.60 m). The two cantilever parts of the beam are used to generate the support bending moment with the vertical jacks  $P_{left}$  and  $P_{right}$  controlling the rotations  $\theta_{left}$  and  $\theta_{right}$ . The horizontal jack  $H$  is used to apply the horizontal displacement  $u$  to the specimen. The jacks  $P$  are used to apply the constant loads in the span.

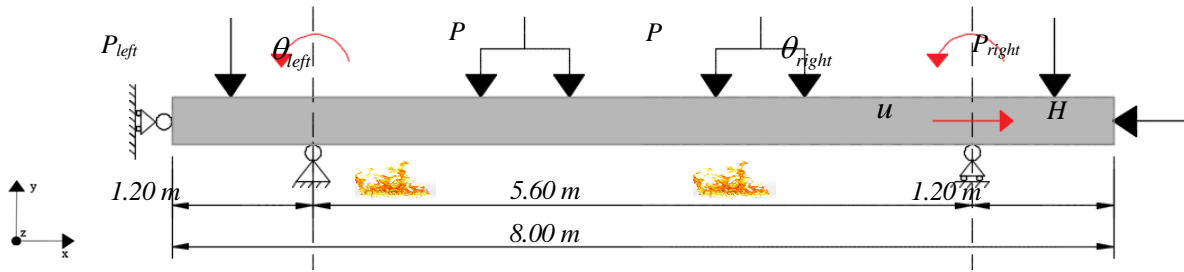


Figure 6. The configuration of the physical specimen PS.

When the PS is extracted from the entire structure, there are six degrees of freedom at the interface. For performing the hybrid fire test, the PS is placed in the furnace while some degrees of freedom are controlled at the interface. A distinction has to be made between the DoFs that are freed at the interface when cutting the structure (referred to as global) and the DoFs controlled in the furnace (referred to as local). The selection of the local DoFs is influenced by the structure configuration but also by the specificities of the furnace facility and the need to avoid rigid body modes. In this case, three out of the six global DoFs are blocked in the furnace to avoid rigid body motion: the horizontal displacement on the left support, the vertical displacements of the right and left support. The controlled DoFs during the hybrid fire test are the horizontal displacement on the right support and the two support rotations.

The axial force and the support bending moments vary significantly with the variation of temperature throughout the test, whereas the variation of shear force remains limited in this example. This is the reason why the rotations and the axial displacement are controlled during the hybrid fire test. Nevertheless, the influence of the vertical displacements on the supports is taken into account when the predetermined matrix of the NS is computed. Sauca (2016, pg. 141-145) presents how the condensation from six global degrees of freedom to the three local degrees of freedom is captured in the predetermined matrix.

### The first generation method

The first generation method is sensitive to the stiffness ratio between the NS and PS. For this specific case study, the stiffness ratio is smaller than 1 for the axial DoF and bigger than 1 for the rotational DoFs. The force control procedure is thus requested for the axial DoF and the displacement control procedure is suitable for the rotational DoFs. The force control procedure was used here.

The communication between the PS (modelled in SAFIR) and the NS (predetermined matrix defined in Excel file) was done manually.

Figure 7 presents the evolution in time of interface forces and displacement/rotations when the first generation method is applied in a virtual environment. Every graph illustrates the correct solution along with the interface solutions of the PS and NS. The correct solution represents the solution calculated when the building structure is numerically modeled as an entity (no substructuring).

Soon after the beginning of the virtual hybrid fire test, the solution oscillates around the correct solution leading to an early failure of the beam. The exercise is repeated with different time steps, e.g.

2 s, 20 s and 50 s, leading to instability every time. Figure 7 presents the solution when the time step is equal to 2 s.

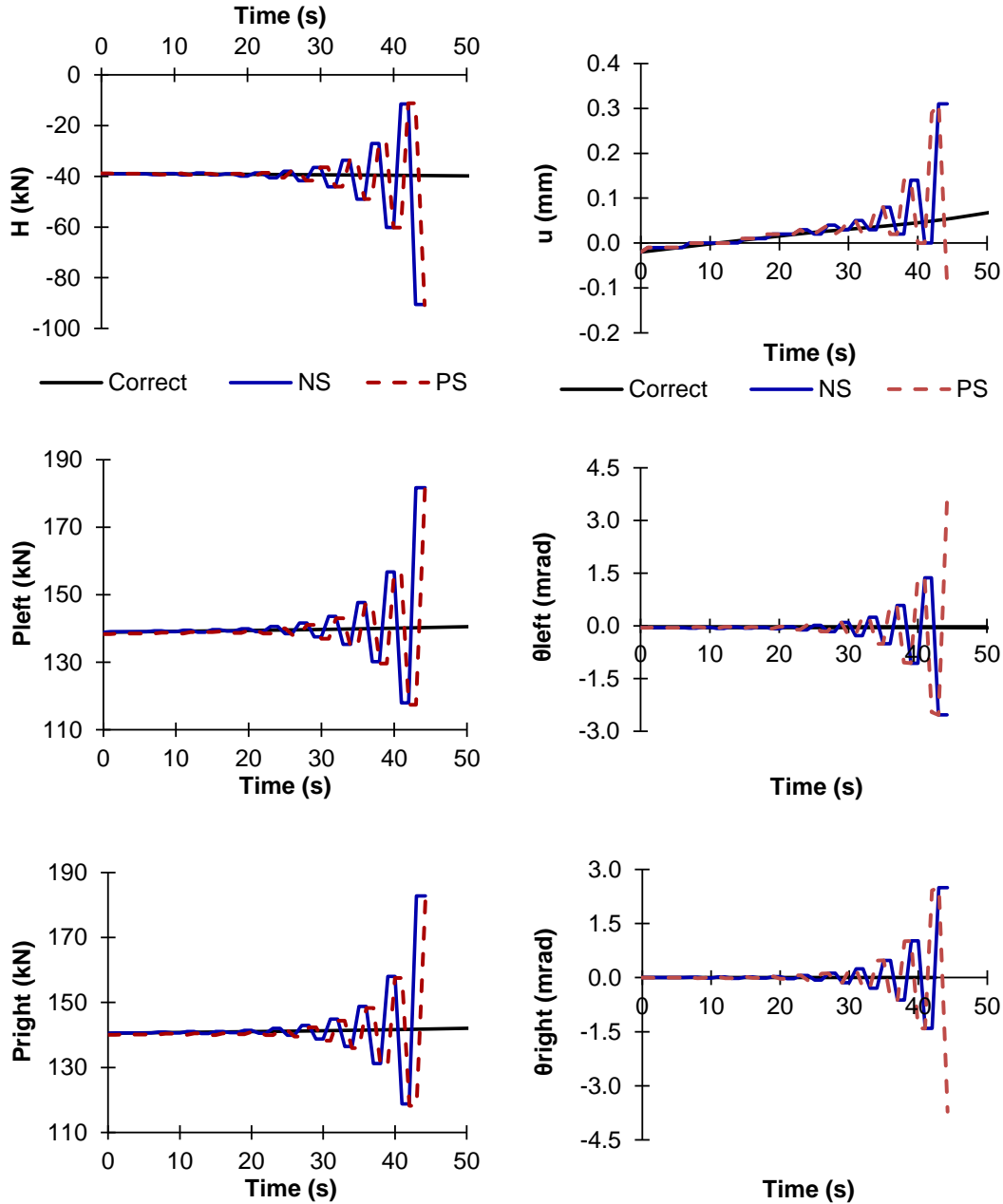


Figure 7. Evolution of interface forces and displacements when using the first generation method in force control procedure

This example supports the conclusions that the first generation method is sensitive to the stiffness ratio between the substructures (Sauca et al., 2016a).

### The new method (second generation method)

In the previous section, the exchange of the information between the PS and NS was done manually for each time step. This tedious exercise could be performed only because instability occurred early in the process.

An automatic procedure was developed in SAFIR in order to perform virtual hybrid fire tests (Sauca, 2016, pg. 150 - 154) when the behavior of the NS is represented by using the predetermined matrix. The HFT subroutine computes the interface displacements to be imposed at the interface of the substructures at every time step.

Figure 8 presents the evolution in time of the interface forces and displacements when the new method is applied in the displacement control procedure.

In a real hybrid fire test, selection of the time step is critical; it must be short enough to ensure convergence while, on the same time, it must be long enough to allow the calculations related to the NS to be performed and the target values to be applied by the actuators. The value chosen for the stiffness of the PS is also a key factor for success. In parallel to a theoretical framework that could guide these choices, the possibility to perform HFT in a virtual environment allows testing the influence of different choices beforehand (considering that some hypotheses have to be made on the behavior of the PS during the test).

In this specific example, the time step of  $1\text{ s}$  is considered while the stiffness of the PS was taken as the initial tangent stiffness multiplied by 1.50.

Six graphs are presented where 3 of them illustrate the evolution of interface forces in time, while the other 3 present the evolution of the interface displacements in time.

Every graph illustrates the “correct solution” along with the solution registered at the interface of the PS and the solution computed at the interface of the NS. The “correct solution” results from the numerical analysis of the structure when no substructuring is done. The interface solutions of the PS result from the virtual hybrid fire testing analysis. The NS is defined using a constant matrix and the interface solution is computed during the virtual hybrid fire test.

In addition to the information presented in Figure 8, Figure 9 presents the force-displacement and force-rotation diagrams.

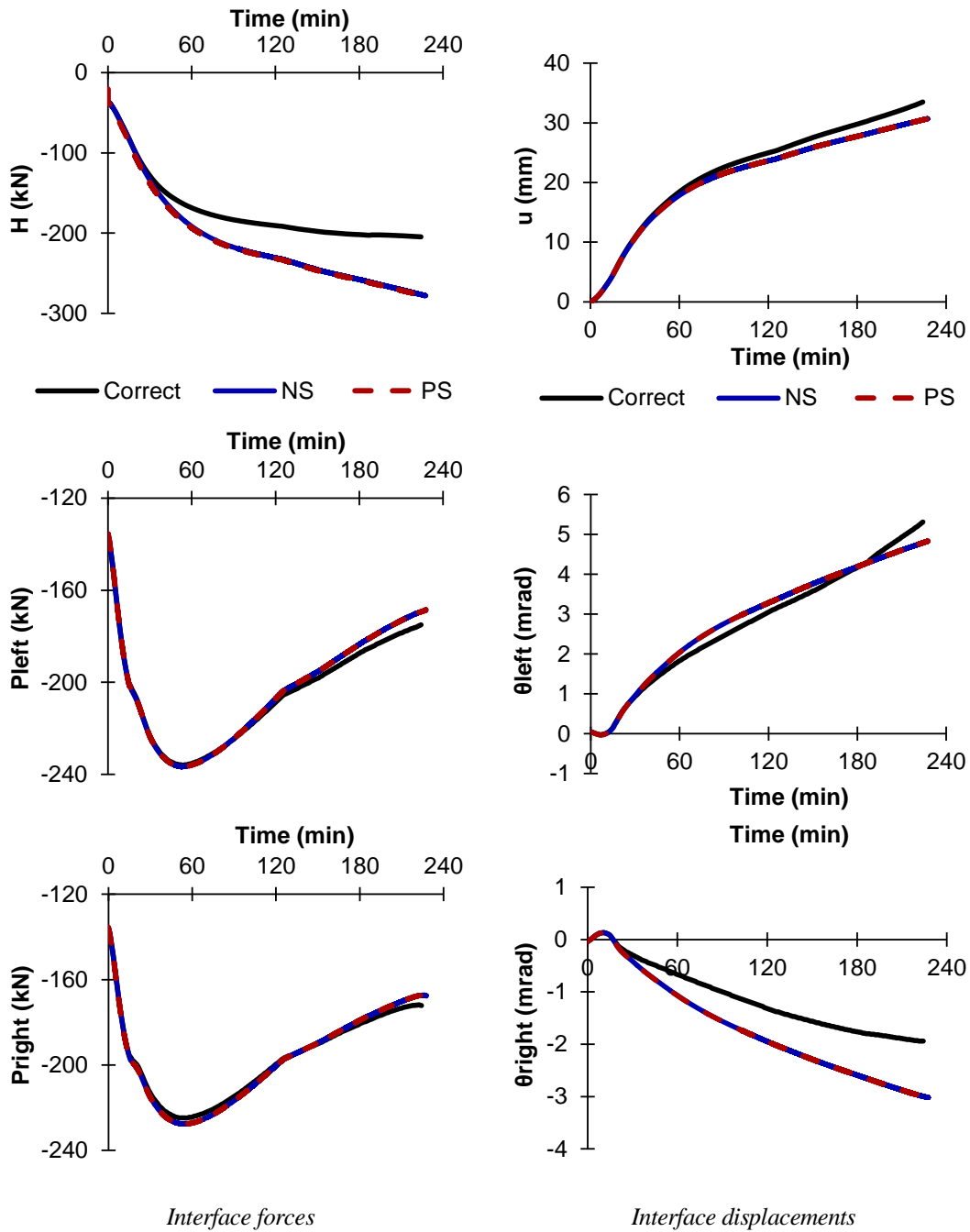


Figure 8. Evolution of interface displacements and forces in time when using the new method in displacement control procedure

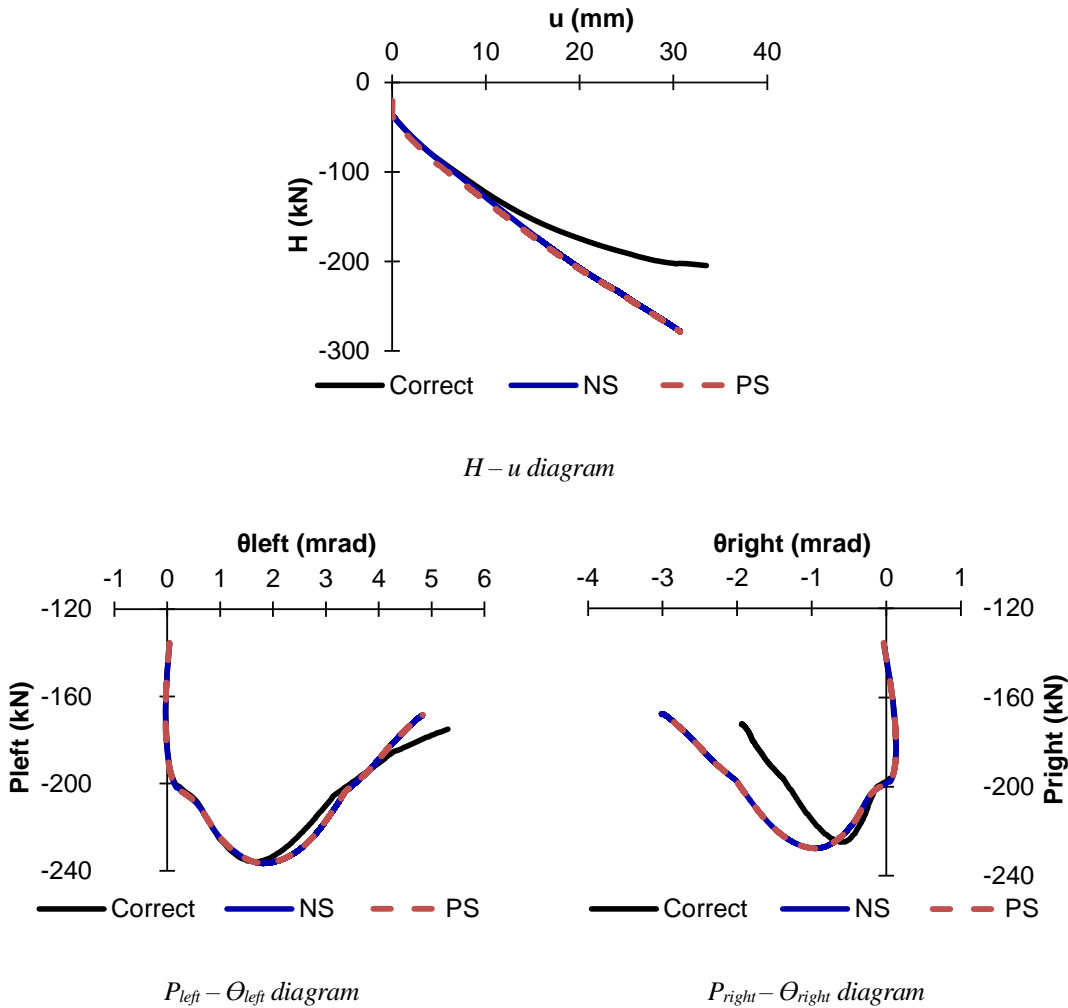


Figure 9. Force-displacement and force-rotations diagrams when using the new method in displacement control procedure

No instability occurs when using the new method of HFT.

One criterion of a successful HFT is when equilibrium and compatibility are ensured between the substructures during the entire process. Figure 8 and Figure 9 show that this is the case in this application, since the solution measured at the interface of the PS coincides with the solution of the NS.

Another criterion is that the “correct” solution needs to be reproduced during the test. In a real test, this correct solution is not available because it cannot be envisaged to make a test on the complete structure. On the contrary, the correct solution can be calculated in a virtual environment; because of that, a virtual hybrid testing environment is a valuable tool for testing different methodologies or the influence of different parameters.

In this particular example, Figure 8 shows that the solution yielded by the hybrid test diverges somehow from the correct solution after 30 minutes. This is due to the fact that the behavior of the NS is represented by a constant matrix in the HFT whereas the correct solution has been computed in SAFIR considering a geometrically and materially nonlinear behavior. Indeed, the time when both solutions start to diverge corresponds to the time when a nonlinear material behavior develops in the NS. To further validate this assumption, the simulation was repeated for the case of an elastic NS. In this case, the “correct solution” corresponds to the numerical analysis of the entire structure using a linear elastic material for the structure around the beam. Figure 10 shows that the solution computed by means of HFT reproduces exactly the “correct solution” (the correct, PS and NS curves lie on top of each other). The time step and the stiffness of the PS used in the calculations are the same as in the case of the nonlinear NS. These results prove that, when the NS behavior is linear, it can be modeled accurately in the HFT using a constant predetermined matrix. However, when the NS behavior is nonlinear, the use of a constant predetermined matrix does not lead to a successful reproduction of the correct solution (Figure 8-9). In this case, the nonlinear behavior can be approximated by using multiple matrices (the nonlinear elastic behavior is linearized) or FE models (Sauca, 2016, pg. 42-45).

In addition to the information presented in Figure 10, Figure 11 presents the force-displacement and force-rotation diagrams for the case when the NS is elastic.



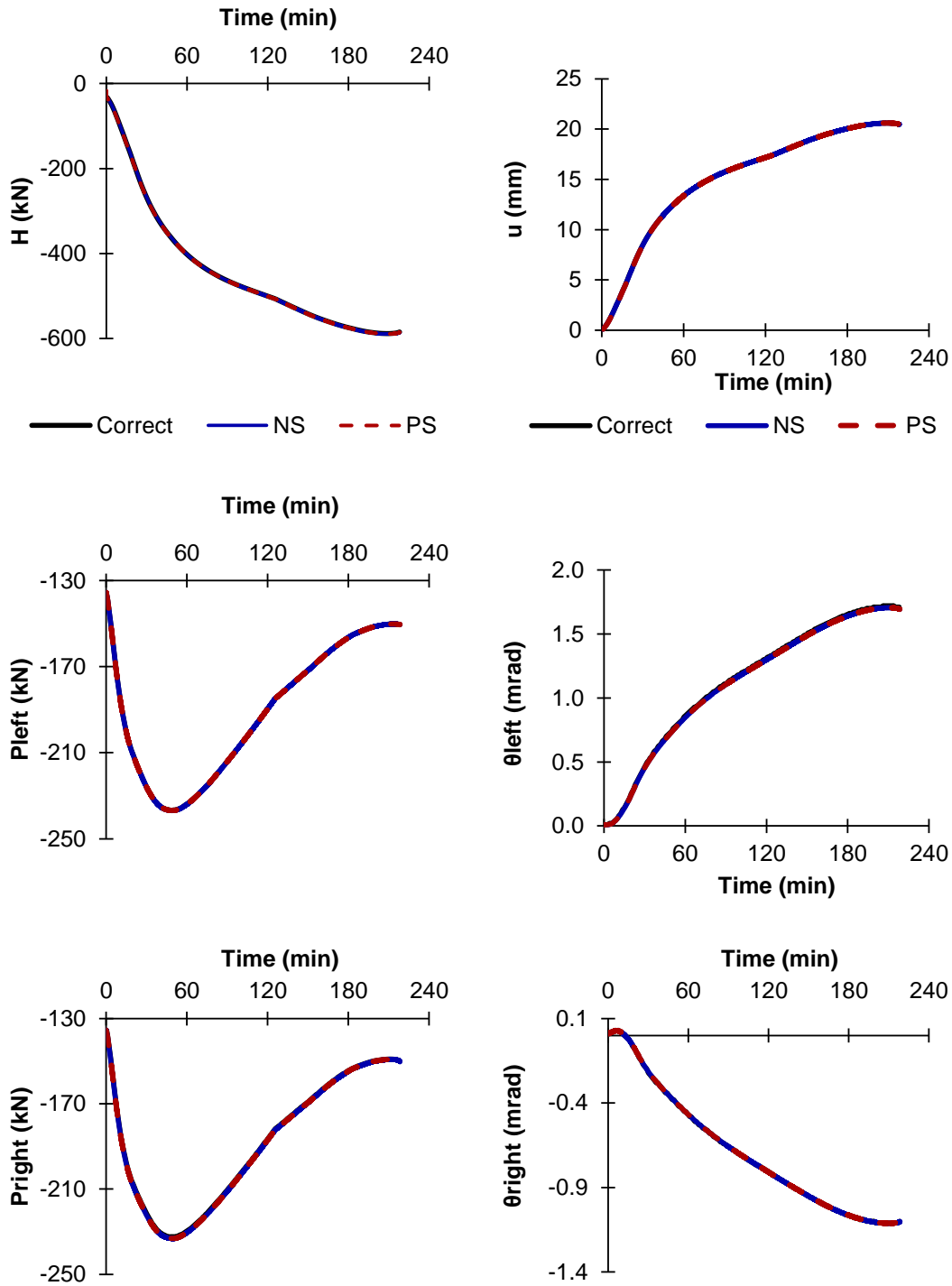


Figure 10. Evolution of interface displacements and forces in time when using the new method in displacement control procedure, for a linear elastic NS

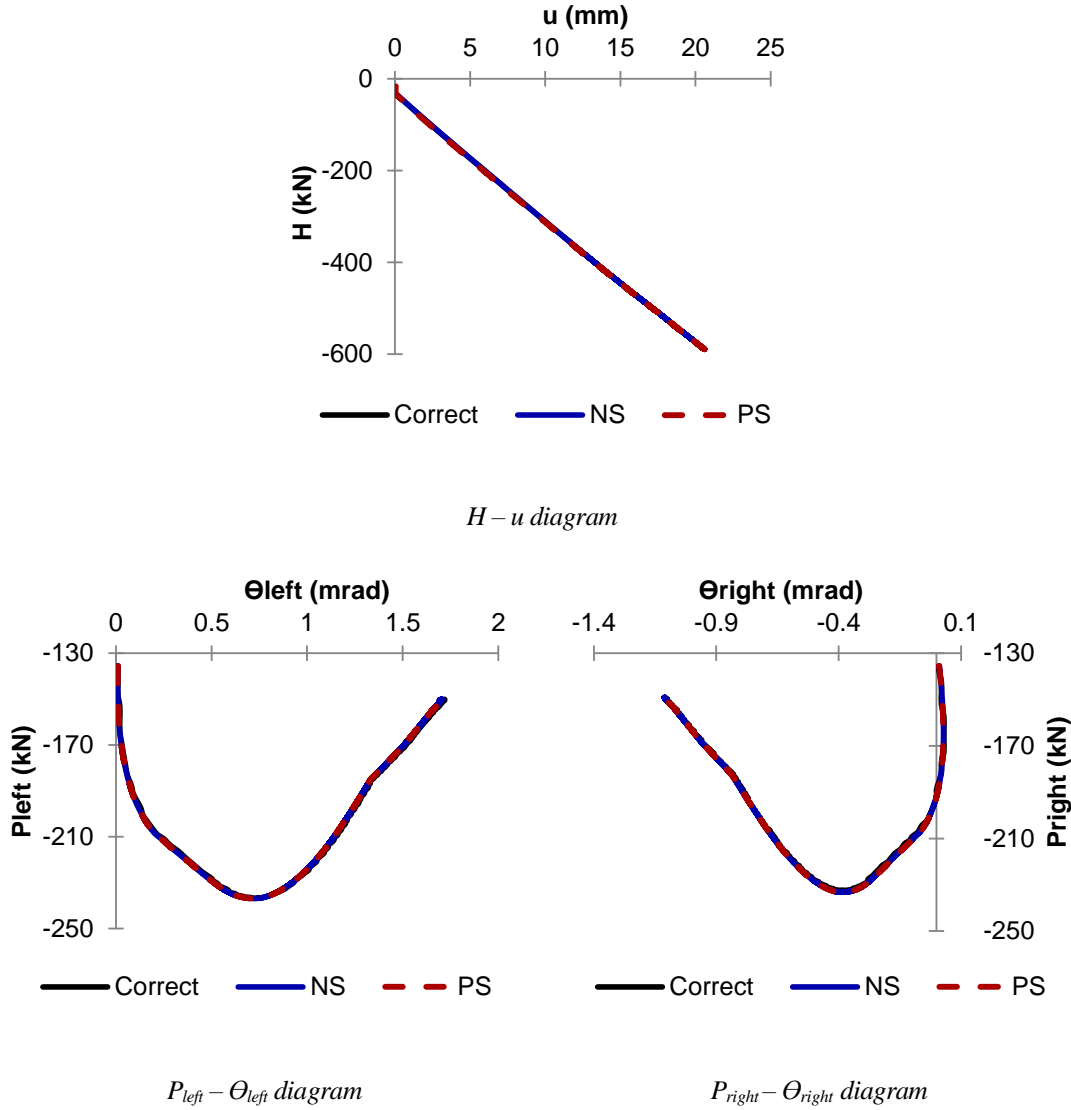


Figure 11. Force-displacement and force-rotations diagrams when using the new method in displacement control procedure, for a linear elastic NS

## 6. CONCLUSION

The objective of the paper was to show that the first generation HFT method presented in the literature, where the correction of the interface forces/displacements depends only on the characteristics of the NS, is not systematically stable. It has been shown, using a linear elastic system as illustrative example, that the stiffness ratio between the NS and PS will dictate the stability of this method. Yet, the stiffness ratio is not easily predictable before a fire test, because the stiffness of the exposed substructures is reduced during the test. Moreover the need of controlling multiple DoFs makes the method impossible to be applied, when different types of procedure should be used for different DoFs.

A new method has been proposed in this paper, unconditionally stable no matter the stiffness ratio, and illustrated in a displacement control procedure.

Virtual hybrid fire testing, when both substructures are modeled numerically, was performed in the case of two examples, namely a simple elastic system and a reinforced concrete moment resisting frame. In both examples, the first generation method lead to instabilities in the response, whereas the new methodology succeeded in ensuring stability, equilibrium and compatibility throughout the tests.

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